
#### Abstract

Aims This policy outlines the teaching, organisation and management of the mathematics taught and learnt at Hillsgrove Primary School. The school's policy for mathematics is based on the document 'Framework for teaching mathematics from Reception to Year 6.' The implementation of this policy is the responsibility of all the teaching staff.


## Equal Opportunities

We recognise that all children come to school with a range of experiences and proficiencies, and that in order to learn with the mathematics curriculum these should not be disregarded but recognised and developed. Pupils who have a heritage language may explore or write numbers in a different formation.

## Teaching Mathematics

## Planning

The National Strategy plans are used to ensure coverage and progression of all topics. Topics are not always followed in the order given and changes are made within the term due to resources or based on the children's needs.

## Teaching time

To provide adequate time for developing numeracy skills each class teacher will provide a daily mathematics lesson. This may vary in length but will usually last for 50 to 60 minutes. Links will also be made to mathematics within other subjects so pupils can develop and apply their mathematical skills.

## Class Organisation

From Reception onwards, all pupils will have a dedicated daily mathematics lesson. Within these lessons there will be a good balance between whole-class work, group teaching and individual practice.

## A typical lesson

A typical 50 to 60 minute lesson in Reception to 6 will be structured like this:

- Oral work and mental calculation (about 5 to 10 minutes)

This will involve whole-class work to rehearse, sharpen and develop mental and oral skills.

- The main teaching activity (about 30 to 40 minutes)

This will include both teaching input and pupil activities and a balance between whole class, grouped, paired and individual work.

- A plenary (about 10 to 15 minutes)

This will involve work with the whole class to sort out misconceptions, identify progress, to summarise key facts and ideas and what to remember, to make links to other work and to discuss next steps.

## Practical work

At Hillsgrove we give practical work a high focus. Lessons should include as much practical work as possible, and when appropriate, with a variety of resources being used.

## Mathematical vocabulary

The daily maths lesson will include the proper use of maths terms and vocabulary to help children understand mathematical ideas. Mathematical terms and vocabulary will also be explicitly taught.

## Out-of-class work and homework

The daily mathematics lessons will provide opportunities for children to practice and consolidate their skills and knowledge, to develop and extend their techniques and strategies, and to prepare for their future learning. These will be extended through out-of-class activities or homework. These activities will be short and focused and will be referred to and valued in future lessons.

## Links between mathematics and other subjects

Mathematics contributes to many subjects within the primary curriculum and opportunities will be sought to draw mathematical experience out of a wide range of activities. This will allow children to begin to use and apply mathematics in real contexts.

## School and Class Organisation

## Catering for more able pupils

Where possible more able pupils will be taught with their own class and stretched through differentiated group work and extra challenges. When working with the whole class, teachers will direct some questions towards the more able to maintain their involvement. Very occasionally special arrangements will be made for an exceptionally gifted pupil e.g. they may be taught with children from a higher age range or may follow an individualised programme with more challenging problems to tackle.

## Catering for pupils with particular needs

The daily mathematics lesson is appropriate for almost all pupils. Teachers will involve all pupils through differentiation.

Pupils with special educational needs and individual education plans
Teachers will aim to include all pupils fully in their daily mathematics lessons. All children benefit from the emphasis on oral and mental work and participating in watching and listening to other children demonstrating and explaining their methods. However a pupil whose difficulties are severe or complex may need to be supported with an individualised programme in the main part of the lesson.

## Foundation stage classes

In the Foundation stage, classes will be organised to promote social skills and the development of mathematical language and understanding. Opportunities for children to learn in relevant and interesting contexts will be planned e.g through stories, dice games

## Resources

All classrooms have a supply of basic maths equipment e.g number tracks and squares, digit cards, place value cards, rulers etc. All other maths equipment is centrally stored.

## Information and Communication Technology

ICT will be used in various ways to support teaching and motivate children's learning. ICT will involve the computer, calculators, and audio-visual aids. They will however only be used in a daily mathematics lesson when it is the most efficient and effective way of meeting the lesson objectives.

## Assessment

Assessment will take place at three connected levels: short-term, medium-term and long-term. These assessments will be used to inform teaching in a continuous cycle of planning, teaching and assessment.

Short-term assessments will be an informal part of every lesson to check their understanding and give you information, which will help you to adjust day-to-day lesson plans.

Medium-term assessments will take place in the two 'assess and review' lessons timetabled each half term and will assess some of the ideas linked the key objectives that have been covered during the half term.

Long-term assessments will take place towards the end of the school year to assess and review pupils' progress and attainment. These will be made through compulsory National Curriculum mathematics tests for pupils in Years 2 and 6 and supplemented by the optional QCA tests. Teachers will also draw upon their class record of attainment against key objectives and supplementary notes and knowledge about their class to produce a summative record. Accurate information will then be reported to parents and the child's next teacher.

## Management of Mathematics

## Role of the Coordinator

- Teach demonstration lessons
- Ensure teachers are familiar with the Framework and help them to plan lessons
- Lead by example in the way they teach in their own classroom
- Prepare, organise and lead INSET, with the support of the Head of school
- Work co-operatively with the SENCO
- Observe colleagues from time to time with a view to identifying the support they need
- Attend INSET provided by LEA numeracy consultants
- Inform parents
- Discuss regularly with the head of school and the numeracy governor the progress of implementing the Strategy in the school.


## Role of the Headteacher

- Lead, manage and monitor the implementation of the Strategy, including monitoring teaching plans and the quality of teaching in classrooms
- With the Numeracy governor, keep the governing body informed about the progress of the Strategy
- Ensure that mathematics remains a high profile in the school's development work
- Deploy support staff to maximise support for the Strategy


## Progression towards a Standard Written method of Calculation

The National Numeracy Strategy provides a structured and systematic approach to teaching number. There is a considerable emphasis on teaching mental calculation strategies. Up to the age of 9 (Year 4) informal written recording should take place regularly and is an important part of learning and understanding. More formal written methods should follow only when the child is able to use a wide range of mental calculation strategies.

## REASONS FOR USING WRITTEN METHODS

- To aid mental calculation by writing down some of the numbers and answers involved
- To make clear a mental procedure for the pupil
- To help communicate methods and solutions
- To provide a record of work to be done
- To aid calculation when the problem is too difficult to be done mentally
- To develop and refine a set of rules for calculations


## WHEN ARE CHILDREN READY FOR FORMAL WRITTEN CALCULATIONS?

## Addition and subtraction

- Do they know addition and subtraction facts to 10,20 and 100 and can they apply facts to calculations?
- Do they understand place value and can they partition numbers?
- Can they add three single digit numbers mentally?
- Can they add and subtract any pair of two digit numbers mentally?
- Can they explain their mental strategies orally and record them using informal jottings?


## Multiplication and division

- Do they know multiplication and related division facts up to $10 x$
- Do they know the result of multiplying by 0 and 1 ?
- Do they understand 0 as a place holder?
- Can they multiply two and three digit numbers by 10 and 100 ?
- Can they double and halve two digit numbers mentally?
- Can they use multiplication facts they know to derive mentally other multiplication facts that they do not know?
- Can they explain their mental strategies orally and record them using informal jottings?
- Can they partition by multiples of the divisor - see division- and by place value

If children cannot access age appropriate objectives, track back to previous stages, as necessary. If children are working beyond expectations then move forwards to the next stage.

It is essential that children's mental methods in all four operations are very secure and they are able to use a variety of strategies as appropriate.

## Using the Policy

Each stage corresponds to the same year group. We have used stages to ensure consistency in progression throughout the school and encourage tracking back and forwards depending on a pupils' conceptual understanding, their mathematical skills and their knowledge and use of facts and vocabulary.

## Related objectives:

- Facts
- Place Value
- Understanding

These are the areas that are closely linked to understanding calculation for any particular year group. These objectives need to be taught as the main part of the lesson but should also be part of an on-going programme of mental and oral starters that support the teaching and learning of each of the four operations

## Differentiation

Progression in calculations: The progression details the calculations appropriate for that stage, with some room for extension. Children's fluency in calculation should be given greater emphasis than their ability to use an informal or formal written method. This is particularly important for Year 4 and Year 5 teachers who may be tempted to move the children on to an informal method before they are fluent in a range of strategies in mental calculation.

Strategies: The strategies that children should be aware of are detailed for each stage. Children should be encouraged to use a range of strategies and to consider the most appropriate strategy for any given calculation. Children's ability to consider a range of strategies should be given greater emphasis than their ability to use either an informal or formal written method. This is particularly important for Year 4 and Year 5 teachers who may be tempted to move the children on to an informal method before they are fluent in a range of strategies in mental calculation.

Models, images and resources: These facilitate access to strategies. They are a visual and concrete image to support teachers' explanations for any given strategy e.g. jottings and empty number lines.

Children can then use these models to support their thinking and aid calculation. Children should be encouraged to become as efficient as possible in their jottings until they no longer need to record to support their thinking. They may then move on to more challenging calculations where they do need to use jottings to aid calculation.
E.g. 16+7

Child A - uses fingers to count on 7 from 16;
Child $B$ - uses a number track to demonstrate jumping 4 from 16 to 20 and then on 3 to 23;
Child C - uses an empty number line to jump 4 from 16 to 20 and then on 3 to 23;
Child D - can calculate by splitting 7 into 4 and 3 , quickly, without using jottings.
It is important that teachers are aware of children's strategies and their fluency when
diagnosing their level of competency in calculation, as well as considering whether they got the answer correct.

## Addition

## Stage 1

Related objectives: Facts, Place Value and Understanding(Mental/ Oral starters)

- Within the range of $0-50$ or beyond say the number that is 1 or 10 more or less than any given number
- Know by heart all pairs of numbers with a total of 10
- To recognise + and = signs in simple number sentences
- To partition at least a teens number
- To understand addition can be done in any order
- Understand and use related vocabulary: more, add, sum, total, altogether, equals, sign


## Progression in calculations

- $U+U$
- teen numbers $+U$
- 2 digit + U
- 2 digit $+U$ crossing the tens boundary
- teen numbers + teen numbers


## Strategies

- Combining two sets counting all
- Put larger number first and find total by counting on
- Use partitioning for a teen number + teen number


## Models and Images

- Using counters, blocks, fingers and bead string
- Use laminated number lines and hundred squares to aid calculation by drawing jumps to show the addition
- $6+6=12$



## Stage 2

## Related objectives (Mental/ Oral starters)

## Facts and Place Value

- Know what a 2 digit number represents, including 0 as a place holder, and partition a 2 and then 3 digit numbers into a multiple of (100) 10 and 1s
- Say the number that is 1 or 10 (100) more or less than any given 2 digit number
- Know all addition and subtraction facts for each number to 10* and then 20
- Know bonds of multiples of 10 to 100


## Understanding

- Subtraction is the inverse of addition ( $5+_{-}=21 \quad 21-5=16$ )
- Know that addition can be done in any order
- Understand and record using + and = sign
- Understand and use related vocabulary: more, add, sum, total, altogether, equals, sign


## Progression in calculations

- $16+3$
- 16+7 (e.g. by counting on in ones)
- 20+20
- 20+12
- $14+11$
- $16+7$ (e.g. by splitting 7* (requires knowledge of number bonds) into 4 and 3 )
- 15+16
- $25+19$


## Strategies

- Put larger number first and count on
- Partition, add and recombine
- Count on in tens and ones $43+32=43+10+10+10+1+1=75$
- Use knowledge of bonds to $10(24+8=\mathbf{2 4}+\mathbf{6}+2=30+2=32)$
- Adding near multiple of 10
- Doubles and near doubles (e.g. 6+7, $40+39$ )


## Models and Images (Use any of the models and images below to support the teaching of the strategies above)

1. Use of bead string and number track
2. Use of hundred square to show jumps of $\mathrm{T}, \mathrm{U}, \mathrm{TU}$
3. Simple jottings:
$16+13=16+10+3=29$
4. Blank number lines:
$26+23=26+10+10+3$


## Stage 3

## Related objectives (Mental/ Oral starters)

Facts and Place Value

- Partition 3 digit numbers into multiple of 100, 10 and 1
- Say the number that is 1,10 or 100 more or less than any given 2 or 3 digit number
- Know all addition and subtraction facts for each number to 20
- Begin to know number bonds to 100


## Understanding

- Subtraction is the inverse of addition (12+_=36, 36-12=24)
- Know that addition can be done in any order
- Understand and record using + and = sign
- Understand and use related vocabulary: more, add, sum, total, altogether, equals, sign


## Progression in calculations

$20+20$
$20+12$
$14+11$
$16+7$ (splitting 7 into 4 and 3 )
$15+16$
$25+19$
$67+24$

70+50 (crossing 100s boundary)
80+56
86+57
$500+300$
$345+300$
$356+427$

## Strategies

Encourage mental fluency: children should not be over reliant on the hundred square and should partition mentally or visualise the empty number line in their heads for TU+TU/U. For children who still need jottings at this stage, encourage efficient strategies e.g bigger jumps*

- Count on in tens \& ones e.g. $43+32=43+10+10+10+1+1=75$ progressing to $43+30+2$ *
- Use knowledge of bonds to $10(24+8=\mathbf{2 4}+\mathbf{6}+2=\mathbf{3 0}+2=32)$
- Adding near multiple of 10
- Doubles and near doubles e.g. 36+35, 60+70, 18+16


## Models and Images

TU + TU, developing to HTU + TU or HTU + HTU. Use hundred squares, blank number lines and simple partitioning and recombining (as year 2, with more challenge):

1. By counting on in multiples of 10 or $1: 86+57=86+50+7=136+7=143$
2. And multiples of 100,10 or $1: 356+427=356+400+20+7=756+20+7=776+7=783$
3. Adding a near multiple of $10: 35+19=35+20-1$


Use empty number line to demonstrate. Children may calculate mentally or with jottings.

Stage 4
Related objectives (Mental/ Oral starter)
Facts and Place Value

- Partition 4 digit numbers into multiple of 1000, 100, 10 and 1
- Say the number that is $1,10,100$ or 1000 more or less than any given 2,3 or 4 digit number
- Derive quickly all pairs of numbers that total 100 eg. $60+40,75+25,38+62$
- Use knowledge of place value and facts to estimate and check


## Understanding

- Subtraction is the inverse of addition ( $33+==75,75-33=42$ )
- Knowledge that addition can be done in any order
- Understand and record using + and = sign
- Understand and use related vocabulary: more, add, sum, total, altogether, equals, sign, increase, inverse


## Progression in calculations

$20+20$

$$
80+56
$$

20+12
86+57
$14+11$
16+7 (splitting 7)
500+300
$15+16$
25+19
$345+300$

67+24
3200+600
$70+50$ (crossing 100s boundary)

## Strategies (Use models and images as for Stage 3)

- Put larger number first
- Count on in hundreds, tens \& ones e.g. $225+325=325+200+20+5$
- Use knowledge of bonds to $10(24+8=\mathbf{2 4 + 6}+2=\mathbf{3 0}+2=32)$
- Adding near multiple of 10
- Doubles and near doubles (e.g. 38+36, 160+170, 380+380)


## Written Method (if children are struggling to learn this procedure quickly go back to models and images for stage 3)

Introduce informal methods for addition. TU + TU to learn method, adding least significant digit first. As soon as children are secure with this method, extend to HTU + TU, still working in the expanded form.

| 47+76 | 347+276 (to model procedure) | Leading quickly to: |
| :---: | :---: | :---: |
| 40+7 | $300+40+7$ |  |
|  |  | 368 |
| 70+6 | 200+70+6 | + 493 |
| 13 | 13 | 11 |
| 110 | 110 | 150 |
| 123 | 500 | 700 |
|  | 623 | 861 |

## Stage 5

## Related Objectives: Facts, Place Value and Understanding (Mental /Oral Starter)

- Partition 4 digit numbers into multiple of 1000, 100, 10 and 1
- Say the number that is 1, 10, 100 or 1000 more or less than any given 2,3 or 4 digit number
- Count forward and backwards in steps of $0.1,0.2,0.3 \ldots$
- Derive quickly decimals that total 1 e.g. $0.2+0.8,0.75+0.25,0.32+0.68$
- Derive quickly pairs that total 100
- Add several numbers e.g. single digits, or multiples of ten e.g. $40+50+80$
- Develop further relationship between addition and subtraction
- Understand and use related vocabulary: more, add, sum, total, altogether, equals, sign, increase, inverse


## Progression in calculations

| $20+20$ | $500+300$ |
| :--- | :--- |
| $20+12$ | $345+300$ |
| $14+11$ | $356+427$ |
| $16+7$ (spliting 7) | $3200+600$ |
| $15+16$ | $3200+900$ |
| $25+19$ | Decimals |
| $67+24$ | $5.4+2.1$ |
| $70+50$ (crossing 100s boundary) | $6.4+3.8$ |
| $80+56$ | $0.63+0.28$ |
| $86+57$ |  |

## Strategies (Use models and images as for Stage 3)

- Add nearest multiple of 10 or 100 and adjust
- Count on (use empty number line for children finding decimals difficult)
- Partition and use knowledge of number bonds
- Doubles and near doubles


## Written method

Expanded method for HTU+HTU, leading to ThHTU + HTU and decimal numbers.

| 368 |  |
| ---: | ---: |
| +493 |  |
| 11 |  |
| 150 | 2346 <br> 700 <br> 3187 |
| 861 |  |
|  | 120 |
| 400 |  |
|  | 5000 |

If children are secure with this, use compact (carrying) method (NB School must be consistent in method of recording for carrying.):

## Stage 6

Related Objectives: Facts, Place Value and Understanding (Mental/ Order starter)

- Partition 4 digit numbers into multiple of 1000, 100, 10 and 1
- Say the number that is $1,10,100$ or 1000 more or less than any given 2,3 or 4 digit number
- Count forward and backwards in steps of $0.1,0.2,0.3 \ldots$ and 0.25
- Derive quickly decimals that total 1 eg. $0.2+0.8 \quad 0.75+0.25 \quad 0.32+0.68$
- Derive quickly pairs that total 100
- Add nearest multiple of 10 or 100 and adjust
- Develop further relationship between addition and subtraction
- Understand and use related vocabulary: more, add, sum, total, altogether, equals, sign, increase, inverse


## Progression in calculations

20+20
20+12
14+11
$16+7$ (spliting 7)
15+16 (crossing 10s boundary)
25+19
67+24
70+50 (crossing 100s boundary)
80+56
86+57
Add several numbers e.g. single digits or multiples of ten e.g. $40+50+80$

500+300
$345+300$
356+427
3200+600
3200+900
Decimals
$5.4+2.1$
$6.4+3.8$
$0.63+0.28$
Add several numbers with different
number of digits eg $23+139+4+205$

## Strategies (Use Models and Images as for Year 3)

- Add nearest multiple of 10 or 100 and adjust
- Count on partitioning the smaller number e.g. $154+775=775+100+50+4$
- Doubles and near doubles
- Knowledge of number bonds to 10,20 and 100


## Written Method

Use of expanded method for ThHTU + ThHTU, and if children are secure, using compact method (carrying), including decimals

2686
+1385
4071

111

## Subtraction

## Stage 1

## Related objectives

## Facts and Place Value

- Within the range of $0-50$ or beyond say the number that is 1 or 10 more or less than any given number
- Know by heart all pairs if numbers with a total of 10
- To partition at least a teens number


## Understanding

- To recognise and record - and = signs in simple number sentences
- Use knowledge that subtraction is inverse of addition
- Understand \& use related vocabulary: take away, subtract, how many are left, how much less is..than..,difference between, how much more is..than.., how many more to make
- Understand that e.g. $10-3=7,20-3=17,30-3=23$


## Progression in calculations

- 7-1
- 7-3
- 15-3
- 10-3
- 20-6
- 15-10
- 15-7


## Strategies

- Count how many are left after some have been taken away
- Count back on a number line
- Count on to see how many were removed or how many will make a given number


## Models and Images

- Counters, block and bead string
- Use of hundred square and number track/number line

Children need experience of counting back, e.g. 17-4, and of counting on e.g. 12-9.
NB When counting on use vocabulary of how many more? / difference between

## Counting back

17-4


Counting on


## Stage 2

## Related objectives (Mental / Oral Starter)

## Facts and Place Value

- Use known number facts to add and subtract mentally (using jotting where appropriate)
- Know what a 2 digit represents including 0 as a place holder and partition a 2 then a 3 digit number into a multiple of (100) 10 and 1 s ;
- Say the number that is 1 or 10 more or less than any given 2 then 3 digit number;
- Know all addition and subtraction facts for each number to 10
- Recognise when it is easier to count up or back when finding the difference


## Understanding

- Use knowledge that subtraction is inverse of addition ( $-4=6 \quad 6+4=10$ )
- Record mental subtraction using - and = signs
- Use related vocabulary
- Understand \& use related vocabulary: take away, subtract, how many are left, how much less is..than..,difference between, how much more is..than.., how many more


## Progression in calculations

- 20-6
- 15-10
- 15-7 (splitting 7 into 5 and 2)
- 40-20
- 45-20
- 45-23
- 40-19
- 45-27


## Strategies

- Count on or back on empty number lines
- Partitioning
- Bridging through 10 or a multiple of 10 (using knowledge of number bonds for numbers 1-10)


## Models and Images

As year 1, children need experience of counting on and back, using larger numbers, but should be able to decide which strategy is most appropriate, e.g. 23-17: by counting on from 17 to $23,23-6$ : by counting back 6 from 23.

Children should be using number tracks, hundred squares and empty number lines to calculate.
Counting Back
45-20


## Counting on

23-17
0


Children working with bigger numbers may use partitioning method:
e.g. $47-24=$
$47-20=27$
$27-4=23$

This may be carried out mentally or blank number line:

$23 \quad 27$

## Stage 3

## Related objectives (Mental / Oral Starter)

- Partition 3 digit numbers into multiple of 100, 10 and 1
- Say the number that is 1,10 or 100 more or less than any given 2 or 3 digit number
- Know all addition and subtraction facts for each number to 20
- Begin to know number bonds to 100, initially multiples of 10, then all bonds e.g. 62+38


## Understanding

- Use knowledge that subtraction is inverse of addition (_-10=36, 36+10=46)
- Record mental subtraction using - and = signs
- Understand \& use related vocabulary: take away, subtract, how many are left, how much less is..than..,difference between, how much more is..than.., how many more


## Progression in calculations

| $45-23$ | $819-200$ |  |
| :--- | :--- | :--- |
| $15-7$ | $1200-500$ |  |
| $40-19$ | $900-7$ |  |
| $45-27$ | $4000-3$ |  |
| $120-30$ | $905-7$ |  |
| $146-50$ | $4641-3$ |  |
| $800-500$ | $372-368$ | (small difference) |
|  |  |  |

## Strategies

- Counting on and back on empty number line in ones, tens and hundreds
- Partitioning


## Models, Images and Resources

100 square, number lines and bead string
TU - TU, developing to HTU - TU, and HTU - HTU
It is important that children should decide which method of subtraction is appropriate for each calculation, and use the blank number line approach (see overleaf)
counting on
45-27


## Counting Back

146-18


Progressing to 256-137 (counting on)


Children should not be introduced to the vertical/column method of subtraction in Year 3.

Related objectives: Facts, Place Value and Understanding (Mental / Oral Starter)

- Use known number facts to add and subtract mentally (using jotting where appropriate)
- Partition 4 digit numbers into multiple of $1000,100,10$ and 1
- Say the number that is $1,10,100$ or 1000 more or less than any given 2,3 or 4 digit number
- Derive quickly all pairs of numbers that total 100 eg, $60 \quad 40 \quad 75 \quad 25 \quad 3862$
- Use knowledge that subtraction is inverse of addition ( $-23=25,23+25=48$ )
- Record mental subtraction using - and = signs
- Understand \& use related vocabulary: take away, subtract, how many are left, how much less is..than.., difference between, how much more is..than.., how many more to make, decrease, inverse


## Progressions in calculations

| $45-23$ | $1200-500$ |  |
| :--- | :--- | :--- |
| $15-7$ | $900-7$ |  |
| $40-19$ | $4000-3$ |  |
| $45-27$ | $905-7$ |  |
| $120-30$ | $4641-3$ |  |
| $146-50$ | $372-368$ | (small difference) |
| $800-500$ | $7003-6899$ |  |
| $819-200$ |  |  |

## Strategies

- Finding a small difference by counting on
- Subtract nearest multiple of ten and adjust
- Use number facts and place value
- Partitioning
- Counting on and back


## Models and Images (As for Year 3)

Using number line efficiently e.g. 256-137


Counting on
754-86


If you feel confident that children are ready, move on to expanded method in Stage 5.
We recommend children should not be using the vertical/column method at this stage.

Children should be encouraged to check calculations using estimation \& inverse.

## Stage 5

Related objectives: Facts, Place Value and Understanding (Mental /Oral Starter)

- Partition 4 digit numbers into multiple of 1000, 100, 10 and 1
- Say the number that is 1,10 or 100 more or less than any given 2,3 or 4 digit number - crossing boundaries e.g. 1000-110, 801-10 etc
- Derive quickly all pairs of numbers that total 100 e.g. $60 \quad 40 \quad 75 \quad 25 \quad 38 \quad 62$
- Count forwards and backwards in steps of 0.1, 0.2, 0.3...
- Derive quickly decimal subtraction facts to 1 e.g. 1 - 0.6
- Use knowledge that subtraction is inverse of addition
- Record mental subtraction using - and = signs
- Understand \& use related vocabulary: take away, subtract, how many are left, how much less is..than..,difference between, how much more is..than.., how many more to make, decrease, inverse


## Progression in mental calculations

| $45-23$ | $4000-3$ |  |
| :--- | :--- | :--- |
| $15-7$ | $905-7$ |  |
| $40-19$ | $4641-3$ |  |
| $45-27$ | $372-368 \quad$ (small difference) |  |
| $120-30$ | $7003-6899 \quad$ |  |
| $146-50$ | $5.7-2.5 \quad$ (decimals) |  |
| $800-500$ | $6.2-3.8$ |  |
| $819-200$ | $0.63-0.48$ |  |
| $1200-500$ | $0.7-0.26$ |  |
| $900-7$ |  |  |

## Strategies (Use Models and Images as for Year 3)

- Partitioning
- Counting on and back (particularly for decimals)
- Find a small difference
- Use number facts and place value


## Written Method

Use the expanded form of decomposition as preparation for the compact method where children are secure. TU-TU, to explain the method then move onto HTU - HTU, ThHTU ThHTU, and decimals if appropriate (some pupils may benefit from continuing to use an empty number line for decimals).
e.g. $754-486=$

| 700 | 50 | 4 | 700 | 40 | 14 |
| ---: | ---: | ---: | ---: | ---: | ---: |
| -400 | 80 | 6 | $-\quad 400$ | 80 | 6 |


| 600 | 140 | 14 |  |  |  |
| ---: | ---: | ---: | ---: | :--- | :--- |
| $-\quad 400$ | 80 | 6 |  |  |  |
| 200 | 60 | 8 |  | 268 |  |

Leading quickly to the formal contracted form of decomposition:

## Stage 6

Related objectives: Facts, Place Value and Understanding_(Mental / Oral Starter)

- Finding a small difference quickly
- Partition 4 digit numbers into multiple of 1000, 100, 10 and 1
- Say the number that is 1,10 or 100 more or less than any given 2 or 3 digit number
- Derive quickly all pairs of numbers that total $100 \mathrm{eg}, 60 \quad 40 \quad 75 \quad 25 \quad 38 \quad 62$
- Count forwards and backwards in steps of 0.1, 0.2, 0.3...and 0.25
- Derive quickly decimal subtraction facts to 1 eg $1-0.6$ and $1-0.75$
- Understand \& use related vocabulary: take away, subtract, how many are left, how much less is..than..,difference between, how much more is..than.., how many more to make, decrease, inverse


## Progression in calculations

| $45-23$ | $4000-3$ |  |
| :--- | :--- | :--- |
| $15-7$ | $905-7$ |  |
| $40-19$ | $4641-3$ |  |
| $45-27$ | $372-368$ |  |
| $120-30$ | $7003-6899 \quad$ (small difference) |  |
| $146-50$ | $5.7-2.5 \quad$ (decimals) |  |
| $800-500$ | $6.2-3.8$ |  |
| $819-200$ | $0.63-0.48$ |  |
| $1200-500$ | $0.7-0.26$ |  |
| $900-7$ |  |  |

## Strategies (Use models and Images as for Year 3)

- Partitioning
- Counting on and back (particularly for decimals)
- Finding a small difference


## Written Method

Formal contracted form of decomposition (as in Stage 5) using larger numbers and decimals.

$$
\begin{array}{rr}
9761 & 76.42 \\
-\quad 7593 & -53.98 \\
\hline 2168 & \underline{22.44} \\
\hline
\end{array}
$$

## Multiplication Stage 2

## Related Objectives: Facts and Understanding (Mental/Oral Starters)

- Count in steps of 3 or 4
- Know by heart multiplication facts for $2 x$ x5 and 10x tables
- Know doubles of numbers to at least 15
- Know doubles of multiples of 5 (up to 50)
- To recognise and use $x$ and = signs
- Understand and use related vocabulary: double, times, multiply, multiplied by, multiple of, lots of, groups of, times as (big, long, wide)


## Progression in Calculations

- Multiply a single digit by 1 or 10 (e.g. $6 \times 10 ; 5 \times 1$ )
- Multiply a single digit by 2, 3, 4 or 5 (e.g. $4 \times 3$ )


## Strategies, Models and Images (ITP Multiplication Facts)

Children are introduced to multiplication as:

1. Repeated addition using numbered laminated number lines, drawing on the jumps:
e.g. $3 \times 2=$

2. Using an array
```
* * 3x2=6
    * * 3 lots of 2
    * * 2 multiplied by 3
```

2x3=6
2 lots of 3
3 multiplied by 2
3. Using fingers $2,4,6$ or 3,6


## Stage 3

## Related Objectives and Strategies (Mental/Oral Starters)

Facts and Place Value

- Know by heart multiplication facts for $2 x, x 3, x 4,5 x, x 6$ and $10 x$ tables
- Know doubles of whole numbers to at least 20
- Know doubles of multiples of 5 (up to 100)
- Know doubles of multiples of 50
- Observe and describe the effect of multiplying by 1,10 and 100 using an OHP calculator and place value grid. Develop patterns.

| $* 1$ | 2 | 3 | 4 |
| ---: | ---: | ---: | ---: |
| 10 | 20 | 30 | 40 |
| 100 | 200 | 300 | 400 |

## Understanding

- Know that multiplication and division are inverse ( $3 x=6 \quad 6 \div 3=2$ )
- Know that halving is the inverse of doubling
- Know that multiplication can be done in any order
- Recognise the $x$ and = sign
- Understand and use related vocabulary: double, times, multiply, multiplied by, multiple of, lots of, groups of, times as (big, long, wide)


## Progression in calculation

- Multiply a single digit by $2,3,4,5,6$ or 10
- Multiply a multiple of 10 by $2,3,4,5,6$ or 10 eg. $20 \times 3,40 \times 5$ *
- Derive division facts from known facts demonstrating an understanding of multiplication e.g. $9 \times 6=10 \times 6-6 ; 9 \times 7=10 \times 7-7 ; 4 \times 8=2 \times 8+2 \times 8 ; 8 \times 5$ $=1 / 2(8 \times 10)$ **
- Multiply a teens number by $2,3,4,5$ or 6 e.g. $13 \times 215 \times 3$ without crossing the tens boundary


## Strategies, Models and Images (See Year 2 for related vocabulary)

1. Using an array (see Stage 2)
2. Repeated addition: e.g. $6 \times 3=6+6+6=3 \times 6=3+3+3+3+3+3$
3. Scaling e.g. Make a tower of 4 cubes, 3 times higher.

4. Pupils should calculate up to $6 x$ mentally. Scaffold this either by using fingers, recording multiples e.g. 6, 12, 18 or blank number lines e.g. $6 \times 3$

5. Using counting stick/empty number line to derive new facts from known facts**
6. Partitioning tens and ones start with teens numbers e.g. $12 \times 3=10 \times 3+2 \times 3=30+6=96$
7. Partitioning 2 digit numbers e.g. $35 \times 6=30 \times 6+5 \times 6$

## Stage 4

## Related objectives (Mental/oral starters)

## Facts

- Know by heart multiplication facts up to $10 \times 10$ and derive division facts
- Know doubles of numbers to 50
- Know doubles of multiples of 10 (up to 500)
- Know doubles of multiples of 100 (up to) to 5000


## Understanding

- Know that multiplication and division are inverse ( $6 x$ _=30 $30 \div 6=5$ )
- Know that halving is the inverse of doubling
- Know that multiplication can be done in any order
- Understand and use related vocabulary: multiply, multiplied by, multiple, product, inverse


## Progression in mental calculations

- Multiply a single digit by single digit
- Multiply a multiple of 10 by U e.g. $20 \times 3,40 \times 5$
- Multiply a 2 digit number by $U$ without crossing the tens boundary eg $23 \times 232 \times 3$
- Multiply 2 digit number by $U$ crossing the tens boundary e.g. $13 \times 5$


## Strategies

- Using any known facts to derive new facts
- Multiply by 10 and 100 by shifting digits
- Multiply by 4 (double and double again)
- Multiply by 5 (times by 10 and then halve)
- Multiply by 20 ( times by 10 and then double)
- Multiply by 9 or 11 (times by 10 and adjust)


## Mental Method with recording

Children should be introduced to a mental method, with jottings, using partitioning:
TU x U
$12 \times 3=(10 \times 3)+(2 \times 3)=30+6=96$ OR
$12 \times 3=30+6=96$ (more efficient recording)
$38 \times 7=(30 \times 7)+(8 \times 7)=210+56=266$ OR
38x7=210+56 (more efficient recording)
If children are secure in their mental calculations, strategies and partitioning then introduce the grid layout (Use Grid Method ITP)
Written Method:

| $x$ | 30 | 8 |  |
| :--- | ---: | ---: | ---: |
| 7 | 210 | 56 | 266 |

## Stage 5

## Related objectives (Mental / Oral Starter

## Facts

- Know by heart multiplication facts up to $10 \times 10$ and derive division facts
- Doubles of multiples of whole numbers to 100
- Doubles of multiples of 10 to 1000
- Doubles of multiples of 100 to 10000
- Know square numbers

Understanding

- Know that multiplication and division are inverse (30x_=300, 300 $\div 30=10$ )
- Know that halving is the inverse of doubling
- Know that multiplication can be done in any order
- Understand and use related vocabulary: multiply, multiplied by, multiple, product, inverse


## Progression in calculations

- Multiply a single digit by single digit
- Multiply a multiple of 10 by U eg. 20x3, 40x 5
- Multiply a 2 digit number by U without crossing the tens boundary eg $23 \times 232 \times 3$
- Multiply 2 digit number by $U$ crossing the tens boundary e.g. $13 \times 5$
- Multiply a 2 then 3 digit number by multiples of 10 and 100 e.g. $23 \times 600$
- Multiply a 3 digit number by U
- Multiply a 3 digit number by TU


## Strategies

- Multiply by 10 or 100 by shifting digits
- Use factors eg. $8 \times 12=8 \times 4 \times 3$
- Multiply by 25 (times by 100 and divide by 4 )
- Multiply by 4 (double and double again)
- Multiply by 5 (times by 10 and then halve)
- Multiply by 20 (times by 10 and then double)
- Double 2 digit numbers by partitioning e.g. double $23=$ double 20 and double 3


## Written Method

Extend multiplication to decimals with one place
HTU $x$ U, leading to TU $\times$ TU, using grid layout only:
e.g. $56 \times 27=$

| $x$ | 50 | 6 |  |
| :--- | ---: | ---: | ---: |
| 20 | 1000 | 120 | 1120 |
| 7 | 350 | 42 | 392 |
|  |  |  | 1512 |

## Related objectives

## Facts

- Know by heart multiplication facts up to $12 \times 12$
- Doubles of multiples of whole numbers to 100
- Doubles of 2 digit numbers (e.g. $3.8 \times 2$ )
- Doubles of multiples of 10 to 1000
- Doubles of multiples of 100 to 10000
- Derive square numbers to $12 \times 12$
- Squares of multiples of 10 e.g. $60 \times 60$ (up to 100)

Understanding

- Know that multiplication and division are inverse (36x_=3600, 3600 $\div 36=100$ )
- Know that halving is the inverse of doubling
- Know that multiplication can be done in any order
- Understand and use related vocabulary: times, multiply, multiplied by, multiple, product, inverse


## - Progression in mental calculation

- Multiply a single digit by single digit
- Multiply a multiple of 10 by $2,3,4,5$ or 10 eg. $20 \times 3,40 \times 5$
- Multiply a 2 digit number by 2, 3, 4 or 5 without crossing the tens boundary eg 23x2 32x3
- Multiply 2 digit number by 2,3 , 4 , or 5 crossing the tens boundary e.g. $13 \times 5$
- Multiply a 2 digit number by $6,7,8$ or 9
- Multiply a 2 then 3 digit number by multiples of 10 and 100 e.g. $23 \times 600$
- Multiply a decimal number by a single digit


## Strategies

- Multiply by 10 or 100 by shifting digits
- Use factors e.g. $8 \times 12=8 \times 4 \times 3$
- Multiply by 25 (times by 100 and divide by 4 )
- Multiply by 4 (double and double again)
- Multiply by 5 (times by 10 and then halve)
- Multiply by 20 ( times by 10 and then double)
- Multiply by 49 and 51 (times by 50 and adjust)


## Written Method

Children using standard compact method

```
    275
X 8
2200
    64
```


## Division <br> Stage 2

Related Objectives: Facts and Understanding (Mental / Oral Starters)

- Count forwards and backwards in steps of 3 or 5 using a number track or 100 square
- To know multiplication and division facts for 2, 5 and 10 times.
- Halve any multiple of 10 (up to 100)
- Divide any 2 digit multiple of 10 by 1 or 10 e.g. $60 \div 10=6$
- To recognise and use $\div$ and $=$ signs
- Understand and use related vocabulary: each, share, halve, divide, left over, divided by, equal groups of


## Progression in calculation

- Divide single digit multiples of 2 and 5 by 2 and 5 e.g. $6 \div 2$
- Divide single digit multiples of $2,3,4$, and 5 by $2,3,4$ or 5 e.g. $6 \div 3$
- Divide a 2 digit multiple of 10 by 1 or 10 e.g. $40 \div 10$
- Divide 2 digit multiples of $2,3,4$, and 5 by $2,3,4$ or 5 e.g. $24 \div 3$


## Strategies, Models and Images

Children need to understand division as sharing and then as grouping (ITP: Grouping) Demonstrate practically (e.g. using OHP counters) $12 \div 2$ sharing (divided into/between)
grouping (divided by)


Sharing: 20 cakes shared between 4 people - give one cake to each person and keep going until all cakes are used. How many cakes does each person have?
Grouping: 24 eggs are packed in boxes of 6 . How many boxes are needed? Take 6 eggs and pack the first box, and continue until there are no eggs left. Count how many boxes were needed.
Make link with multiplication using an array or fingers e.g. 15 $\div 3$
$3,6,9,12,15$, the answer is therefore 5 (as five fingers are showing)


Counting groups of the divisor forwards or backwards (ITP Grouping)


## Related Objectives (Mental / Oral Starters)

- Find a difference by counting on;
- To use number facts and place value to subtract mentally;
- Partition in different ways $46=30+16,20+26$ etc


## Facts

- Know by heart multiplication and division facts for $2 x, x 3, x 45 x, x 6$ and $10 x$ table
- Know halves of even numbers to at least 20
- Know halves of multiples of 10 (up to 100)
- Know halves of multiples 100 up to 1000

Understanding

- Know that multiplication and division are inverse ( $\div 3=2,2 \times 3=6$ )
- Know that halving is the inverse of doubling
- Recognise the $\div$ and = sign
- Observe and describe the effect of multiplying and dividing by 1,10 and 100 using an OHP calculator and place value grid. Develop patterns.

| 1 | 2 | 3 | 4 |
| ---: | ---: | ---: | ---: |
| 10 | 20 | 30 | 40 |
| 100 | 200 | 300 | 400 |

- Understand and use related vocabulary: each, share, halve, divide, left over, divided by, equal groups of, remainder


## Progression in calculation

- Divide 2 digit multiples of $2,3,4,5$ and 6 by $2,3,4,5$ or 6
- Divide a 2 digit multiple of 10 by 1 or 10
- Divide any 3 digit multiple of 100 by 10 or 100
- Find remainders in context


## Strategies, Models and Images

NB The empty number line is a model or image used to demonstrate. Pupils may use jottings or calculate the following in mentally.
$33 \div 3$
$1 \times 3$
$10 \times 3$


Using knowledge of multiplication and division facts to find remainders e.g 46 $\div 6=7 \mathrm{r} 4$


Children may find jottings as detailed in Stage 4 a simpler method of recording

## Stage 4

## Related objectives(Mental/oral starters) <br> Facts

- Know by heart multiplication and division facts for all facts up to $10 \times 10$
- Know halves of even numbers to at least 100
- Know halves of multiples of 10 (up to 500)
- Know halves of multiples of 100 (up to 5000)


## Understanding

- Know that multiplication and division are inverse ( $\div 3=4,3 \times 4=12$ )
- Know that halving is the inverse of doubling
- Recognise the $\div$ and = sign
- Understand and use related vocabulary: share, halve, divided by, divisible by, divided into, factor, quotient, remainder, inverse


## Progression in calculation

- Divide 2 digit numbers by $U$
- Divide a 2 digit multiple of 10 by 1 or 10
- Divide any 3 digit multiple of 100 by 10 or 100
- Give a remainder as a whole number
- Decide whether to round up or round down after division in word problem context:

Rounding up: I have 62 cakes. One box holds 8 cakes.
How many boxes do I need to hold all the cakes?
Rounding down: I have £62. Tickets cost £8. How many people can I take?

- Find eighths by halving quarters
- Divide by 10 and 100 by shifting digits
- Partition to find multiples of the divisor (chunking) $66 \div 5=50 \div 5$ and $15 \div 5$
- Find half by partitioning (place value) e.g. half $56=$ half 50 and half 6


## Models and Images

Using jottings and notes to support and informal recording:
Encourage pupils to use knowledge of multiples of 10 of the divisor.
$84 \div 7$
$70 \quad 14$
$10+212$
Pupils need lots of opportunities to partition by multiples of the divisor.
$126 \div 9$ children should be able to partition quickly into 90 and $36 \quad[90=9 \times 10 \quad 36=9 \times 4]$
$96 \div 6$ children should be able to partition quickly into 60 and 36
[60=6x10 $36=6 \times 6]$
Progressing to
$132 \div 6$ partition into 120 and $12 \quad[120=6 \times 20 \quad 12=6 \times 2]$

## Stage 5

Related objectives (Mental/oral starters)

## Facts

- Consolidate knowledge of multiplication and division facts for all tables to $12 \times 12$
- Derive square numbers up to $12 \times 12$
- Know halves of even numbers to at least 100
- Know halves of multiples of 10 (up to 1000)
- Know halves of multiples of 100 (up to 10000)


## Understanding

- Know that multiplication and division are inverse ( $\div 3=10,3 \times 10=30$ )
- Know that halving is the inverse of doubling
- Recognise the $\div$ and $=$ sign
- Understand and use related vocabulary: share, halve, divisor, divided by, divisible by, divided into, quotient, remainder, inverse, factor, multiple, common factor, common multiple


## Progression in calculations

- Divide any 2 digit number by any single digit number
- Divide a 3 digit number by a single digit number
- Divide a 3 digit number by a 2 digit number
- Divide a 3 digit multiple of 10 by 1 or 10
- Divide any 4 digit multiple of 100 by 10 or 100
- Decide whether to round up or round down after division in word problem
- Interpret the calculator display
- To begin to give the quotient (answer) as a decimal and a fraction
$43 \div 4=103 / 4$
$61 \div 4=15.25$
$£ 5.40 \div 4=£ 1.35$


## Strategies

- Find quarters by halving halves
- Find eighths by halving quarters
- Find sixths by halving thirds
- Divide by 10 and 100 by shifting digits
- Partition to find multiples of the divisor (chunking) $66 \div 5=50 \div 5$ and $15 \div 5$
- Find half by partitioning (place value) e.g. half $156=$ half 100 , half 50 and half 6
- Use factors e.g. $90 \div 6=15$ using $3 \times 2=6 \quad 90 \div 3=30 \quad 30 \div 2=15$


## Written Method

Children can continue to use informal method from Stage 4 both when dividing by $U$ and TU. Children can also continue to record using known facts. Alternatively pupils can record it more formally using the chunking method.

$$
\begin{aligned}
& 222 \div 3= \\
& \text { I know } 3 \times 7=21 \therefore \\
& 3 \times 70=210 \\
& 222-210=12 \\
& \text { I know that } 4 \times 3=12 \\
& 70+4=74
\end{aligned}
$$

222

OR $\quad \frac{-210}{12}(\underline{70} \times 3)$
$-\quad 12(4 \times 3)$

## Stage 6

## Related objectives (Mental/oral starters)

## Facts

- Know by heart multiplication and division facts for up to $12 x$
- Know halves of even numbers to at least 100
- Know halves of multiples of 10 (up to 1000)
- Know halves of multiples of 100 (up to 10000)
- Know square root of numbers to $12 x 12$


## Understanding

- Know that multiplication and division are inverse ( $\div 3=36,36 \times 3=108)$
- Know that halving is the inverse of doubling
- Recognise the $\div$ and = sign
- Understand and use related vocabulary: share, halve, divisor, divided by, divisible by, divided into, quotient, remainder, inverse, factor, multiple, common factor, common multiple


## Progression in calculations

- Divide any 2 digit number by any single digit number
- Divide a 3 digit number by a single digit number
- Divide a 3 digit number by a 2 digit number
- Divide a 3 digit multiple of 10 by 1 or 10
- Divide any 4 digit multiple of 100 by 10 or 100
- Decide whether to round up or round down after division in word problem
- Interpret the calculator display
- To begin to give the quotient (answer) as a decimal and a fraction
$43 \div 4=103 / 4$
$61 \div 4=15.25$
$£ 5.40 \div 4=£ 1.35$
- Use decimals to up 2 places


## Strategies

- Find quarters by halving halves
- Find eighths by halving quarters
- Find sixths by halving thirds
- Divide by 10 and 100 by shifting digits
- Partition to find multiples of the divisor (chunking) $66 \div 5=50 \div 5$ and $15 \div 5$
- Find half by partitioning (place value) e.g. half $156=$ half 100 , half 50 and half 6
- Use factors e.g. $378 \div 21=18$ using $3 x 7=21 \quad 378 \div 3=126 \quad 126 \div 7=18$


## Written Method

Pupils can continue to use methods from Satge 5 and when ready move on to the 'bus stop' method. However it remains more efficitent to use chunking or informal jottings when $\div$ TU.

remainder 4
$325 \div 7=$
46
7325

